

SURENDRANAGAR UNIVERSITY

FACULTY OF SCIENCE

Syllabus for

M. Sc. (MATHEMATICS)

With Effect From: 2021-22

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DEPARTMENT OF MATHEMATICS

Course Structure and Scheme of Examination
For Choice based Credit System (CBCS)
(With effect from June-2021)

- Course: **M.Sc. (Mathematics)**
- Eligibility for the admission:- **B.Sc. (Mathematics)**
- Duration:- **Two years**

Programme Outcomes:

- PO1.** Inculcate critical thinking to carry out scientific investigation objectively without being biased with preconceived notions.
- PO2.** Equip the student with skills to analyze problems, formulate an hypothesis, evaluate and validate results, and draw reasonable conclusions thereof.
- PO3.** Prepare students for pursuing research or careers in industry in mathematical sciences and allied fields
- PO4.** Imbibe effective scientific and/or technical communication in both oral and writing. Continue to acquire relevant knowledge and skills appropriate to professional activities and demonstrate highest standards of ethical issues in mathematical sciences.
- PO5.** Create awareness to become an enlightened citizen with commitment to deliver one's responsibilities within the scope of bestowed rights and privileges.

Programme Specific Outcomes

- PSO1.** Understanding of the fundamental axioms in mathematics and capability of developing ideas based on them. Inculcate mathematical reasoning.
- PSO2.** Prepare and motivate students for research studies in mathematics and related fields. Provide knowledge of a wide range of mathematical techniques and application of mathematical methods/tools in other scientific and engineering domains.
- PSO3.** Provide advanced knowledge on topics in pure mathematics, empowering the students to pursue higher degrees at reputed academic institutions.
- PSO4.** Strong foundation on algebraic topology and representation theory which have strong links and application in theoretical physics, in particular string theory.
- PSO5.** Good understanding of number theory which can be used in modern online cryptographic technologies.
- PSO6.** Nurture problem solving skills, thinking, creativity through assignments, project work. Assist students in preparing (personal guidance, books) for competitive exams e.g. NET, GSET, GATE, etc.

Semester 1

Subject Code	Title of the Course	Course Credits	No. of Hrs. Per Week	Weightage For Internal Examination	Weightage For Semester End Examination	Total Marks	Duration Of Semester end Exam in hrs.
CMT – 1001	Algebra 1	4	4	30	70	100	2.5hrs
CMT – 1002	Real Analysis	4	4	30	70	100	2.5hrs
CMT – 1003	Topology 1	4	4	30	70	100	2.5hrs
CMT – 1004	Theory of Ordinary Differential Equations	4	4	30	70	100	2.5hrs
CMT – 1005	Seminar and Problem Session	4	4	-	100	100	-
EMT – 1001	Classical Mechanics 1	4	4	30	70	100	2.5hrs
Total		24				600	

Semester 2

Subject Code	Title of the Course	Course Credits	No. of Hrs. Per Week	Weightage For Internal Examination	Weightage For Semester End Examination	Total Marks	Duration Of Semester end Exam in hrs.
CMT – 2001	Algebra 2	4	4	30	70	100	2.5hrs
CMT – 2002	Complex Analysis	4	4	30	70	100	2.5hrs
CMT – 2003	Topology 2	4	4	30	70	100	2.5hrs
CMT – 2004	Methods in Partial Differential Equations	4	4	30	70	100	2.5hrs
CMT – 2005	Seminar and Problem Session	4	4	-	100	100	-
EMT – 2001	Classical Mechanics 2	4	4	30	70	100	2.5hrs
Total		24				600	

Semester 3

Subject Code	Title of the Course	Course Credits	No. of Hrs. Per Week	Weightage For Internal Examination	Weightage For Semester End Examination	Total Marks	Duration Of Semester end Exam in hrs.
CMT – 3001	Prog. In C & Numerical Methods	4	4	30	70	100	3
CMT – 3002	Functional Analysis	4	4	30	70	100	3
CMT – 3003	Number Theory 1	4	4	30	70	100	3
CMT – 3004	Discrete Mathematics	4	4	30	70	100	3
EMT – 3011 OR EMT – 3021	Differential Geometry OR Sp. Theory of Relativity and Tensor Analysis	4	4	30	70	100	3
PMT – 3001	Practical (Comp. Applications)	4	8	-	100	100	3
Total		24				600	

Semester 4

Subject Code	Title of the Course	Course Credits	No. of Hrs. Per Week	Weightage For Internal Examination	Weightage For Semester end Examination	Total Marks	Duration Of Semester end Exam in hrs.
CMT – 4001	Linear Algebra	4	4	30	70	100	3
CMT – 4002	Integration Theory	4	4	30	70	100	3
CMT – 4003	Number Theory 2	4	4	30	70	100	3
CMT – 4004	Graph Theory	4	4	30	70	100	3
EMT – 4011 OR EMT – 4021 OR EMT – 4031 OR EMT – 4041	Financial Mathematics OR General Theory of Relativity & Cosmology OR Commutative Ring Theory OR Introduction to Mathematical Cryptography	4	4	30	70	100	3

M.Sc. SEMESTER 1

Sub. Code: **CMT-1001**

Core Sub. 1: **Algebra- 1**

Course Outcomes:

Upon completion of the course student will be able to

CO1. Understand basic principles of algebraic structures like groups, fields rings and division rings.

CO2. Recognize and understand the concept of Ideals.

CO3. Recognize and understand the concepts of Euclidean domains, Unique factorization domains, polynomial rings as well as Einstein irreducibility criterion.

Unit 1

Basic concepts of group theory:

Group, abelian group, cyclic group, normal subgroup, quotient group, permutation group, Group isomorphism and their properties, Cayley's theorem, Automorphisms of groups.

Unit 2

Direct Products, Finitely Generated Abelian Groups, Invariants of a finite Abelian Groups, Sylow Theorems.

Unit 3

Quick look at basic ring theory:

Euclidean ring, Quotient ring and zero divisors, Ideals, principal ideal, maximal ideal and prime ideal, Homomorphisms of ideals, Sum and Direct Sum of Ideals, Nilpotent and Nil Ideals.

Unit 4

Euclidean domains, Principal Ideal Domains, Unique Factorization Domains and Polynomial Rings over UFD. Polynomial rings over rational field, irreducible polynomials, Einstein irreducibility criterion.

Reference Books:

- 1) P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, **Basic Abstract Algebra**, Second Edition, Cambridge University Press, 1995.
- 2) M. Artin, **Algebra**, Prentice-Hall of India Private Ltd., New Delhi, 1994.
- 3) J. A. Gallian, **Contemporary Abstract Algebra**, Fourth Edition, Narosa Publishing House, New Delhi, 1999.
- 4) N. S. Gopalakrishnan, **University Algebra**, New Age International Private Ltd. Publishers, New Delhi, Sixth Reprint, 1998.
- 5) I. N. Herstein, **Topics in Algebra**, Second Edition, Wiley Pub. , New York, 1975.

M.Sc. SEMESTER 1

Sub. Code: CMT-1002
Core Sub. 2: Real Analysis

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Understand, define algebra of sets.
- CO2.** Define and understand measurable sets and various types of measures.
- CO3.** Define, understand and utilize the concept of differentiation of monotone functions and absolute continuity.
- CO4.** State and prove theorems including Holder's inequality and Minkowski's inequality.

Unit 1

Algebra of sets, σ -algebra of sets, Borel sets, Lebesgue outer measure, Measurable sets and Lebesgue measure, A nonmeasurable set, Measurable Functions, and Littlewood's three principles.

Unit 2

Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, The integral of a nonnegative function, The general Lebesgue integral, and Convergence in measure.

Unit 3

Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, and Absolute continuity.

Unit 4

L^p spaces, The Holder's inequality, The Minkowski's inequality, and Convergence and completeness.

The course is covered by Chapter 1 Section 4, Chapter 2 Section 7, Chapter 3 (full), Chapter 4(full), Chapter 5 (Sections 1 to 4), and Chapter 6 (Sections 1 to 3) from the book Real Analysis by H. L. Royden, Third Edition, PHI Learning Private Limited (2009) New Delhi.

Reference Books:-

1. Real Analysis by N. L. Carothers, Cambridge University Press (2000).
2. Measure Theory and Integration by G de Barra, Wiley Eastern Limited, First Wiley Eastern Reprint (1987).
3. Real Analysis by V. Karunakaran, Pearson (2012).
4. Fundamentals of Real Analysis by S. K. Berberian, Universitext, Springer (1999).
5. An introduction to Measure and Integration by I. K. Rana, Narosa Publishing House, New Delhi.

M.Sc. SEMESTER 1

Sub. Code: **CMT-1003**
Core Sub. 3: **Topology -1**

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Recognize and interpret the topological structures and their characterizations.
- CO2.** Identify and understand the subspace topology and product topology.
- CO3.** Identify and classify the type of topology including quotient topology and metric topology.
- CO4.** Understand and differentiate the hierarchy of the topological spaces and their characterizations.

Unit-1

Topology, Open sets and closed sets, Finer and Coarser topology, Basis for a topology, Simply ordered topology.

Unit-2

Subspace topology, Product topology, Continuous functions, Homeomorphism.

Unit-3

Limit points, Closure, Interior points and interior, Convergent Sequence.

Unit-4

Metric topology, Uniform convergence, Topology of \mathbb{R}^n .

Unit-5

Connectedness, Local connectedness, Components, Path connectedness

The Course is covered by following Chapter 1, 2 and 3(Upto article 25) of Topology-A first course, J. M. Munkres, Printice Hall of India (2000).

Reference Books:-

- 1) Introduction to Topology and Modern Analysis - G. F. Simmons, Tata McGraw Hill edition-2004.
- 2) General Topology by S. Willard, Addison – Wesley Publishing Company (1970).

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M.Sc. SEMESTER 1

Sub. Code: CMT-1004

Core Sub. 4: Theory of Ordinary Differential Equations

Course Outcomes:

Upon completion of the course student will be able to

CO1. Understand the meaning of Ordinary Differential Equations.

CO2. Understand and solve Partial differential equation.

CO3. Identify and solve Gauss hyper geometric equation.

CO4. Understand, identify and solve Cauchy Problem including Charpit's and Jacobi's method.

Unit 1: Linear System of Differential Equations

The existence and uniqueness theorem, Linear Homogenous systems, Linear Non-Homogenous systems, Nonlinear system of first order equations.

Unit 2: Linear System with constant coefficients

The exponential of matrix, Eigen values and eigen vectors of matrices, calculation of fundamental matrix, two dimensional linear systems, some population problems, an electric circuit .

Unit 3: Series solutions of Linear Differential Equations

Review of properties of power series, second order linear equations with analytic coefficients, theorem on solutions in power series, singular points of linear differential equations, solutions about a regular singular point, exceptional cases, the Bessel equation and some properties of Bessel functions, singularities at infinity, irregular singular points with an introduction to asymptotic expansions

Unit 4: Existence theory

Existence of solutions, uniqueness of solutions, continuation of solutions, the non linear simple pendulum, existence theory for system of first order equations and higher order equations, linear systems, dependence on initial conditions.

Unit 5: Laplace Transforms

Linearity, existence theorem, Laplace transform of derivatives and integrals, shifting theorem, differentiation and integraton of transforms, convolution theorem, inverse Laplace transform, solution of Ordinary Differential equations and integral equations.

This course is covered by “**Ordinary Differential Equations**”, First course by R. Brauer and J. A. Nohel, Second edition, Benjamin Inc.

Reference Books:-

- 1) Ordinary Differential Equations by G. Birkoff and G. C. Rota, Second edition, Ginn and Co(1995)
- 2) Introduction to Ordinary Differential Equations by E. A. Coddington, Prentice Hall of India, 1996.
- 3) Elements of Ordinary Differential Equations by M Golom and M. E. Shinks, Second Edition, McGraw-Hill Books Co., 1965.
- 4) Theory and Problems of Differential Equations by F. Ayers, McGraw Hill, 1972.
- 5) Advanced Engineering Mathematics by E. Kreyzig, John Willey and Sons, 2002.



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M.Sc. SEMESTER 1

Sub. Code: EMT-1001

Elective Sub.1: **Classical Mechanics -1**

Course Outcomes:

Upon completion of the course student will be able to

CO1. Understand and describe elementary principles of motion.

CO2. Understand and criticize equations of motion and classify the dynamical systems.

CO3. Derive and utilize Lagrange's equation of motions.

CO4. Identify, understand and solve two body central force problem.

Unit 1: D'Alemberts principle and Lagrange's Equations

- ☐ Conservation theorem for linear momentum and angular momentum for a particle.
- ☐ Conservation theorem for linear momentum and angular momentum for a system of particles.
- ☐ Classification of dynamical system.
- ☐ Constraints.
- ☐ Virtual displacement and principle of virtual work.
- ☐ Generalized force in holonomic system
- ☐ Mathematical expression for principle of virtual work
- ☐ D'Alembert's principle
- ☐ Lagrange's equation for holonomic system
- ☐ Lagrange's equation for conservative non-holonomic system
- ☐ Problems on above topics

Unit 2: Variational principle and Lagrange's equations

- ☐ Variational principle
- ☐ Calculus of variations
- ☐ Hamilton's principle
- ☐ Derivation of Hamilton's principle from Lagrange's equation
- ☐ Derivation of Lagrange's equations from Hamilton's principle
- ☐ Cyclic co-ordinates
- ☐ Conservation theorems
- ☐ Problems on above topics

Unit 3: Two Body Central force problem

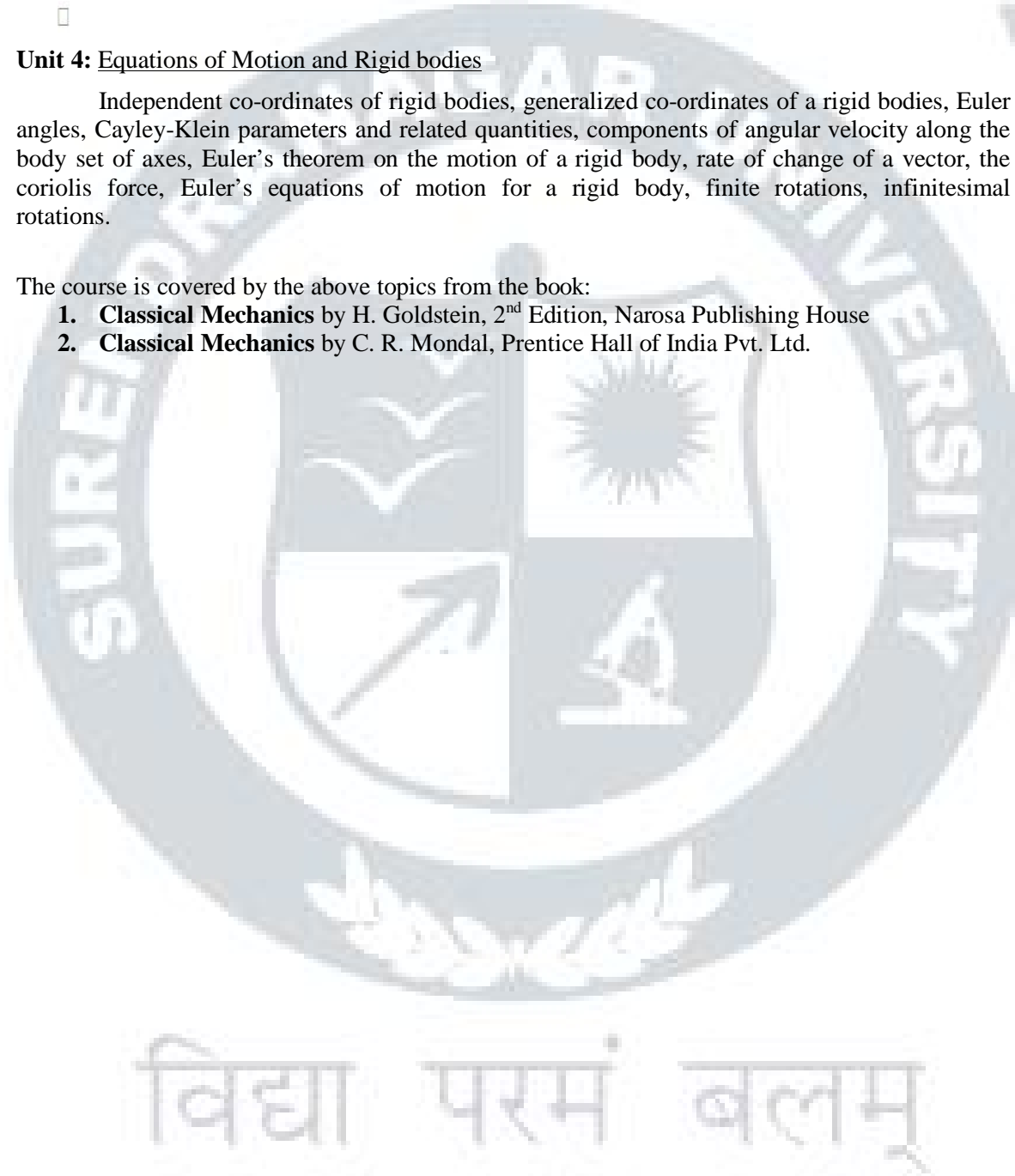
- ☐ Reduction to equivalent one body problem
- ☐ The equations of motion and first integrals
- ☐ The equivalent one dimensional problem and classification of orbits
- ☐ The inverse square law of force.
- ☐

Unit 4: Equations of Motion and Rigid bodies

Independent co-ordinates of rigid bodies, generalized co-ordinates of a rigid bodies, Euler angles, Cayley-Klein parameters and related quantities, components of angular velocity along the body set of axes, Euler's theorem on the motion of a rigid body, rate of change of a vector, the coriolis force, Euler's equations of motion for a rigid body, finite rotations, infinitesimal rotations.

The course is covered by the above topics from the book:

1. **Classical Mechanics** by H. Goldstein, 2nd Edition, Narosa Publishing House
2. **Classical Mechanics** by C. R. Mondal, Prentice Hall of India Pvt. Ltd.



M.Sc. SEMESTER 2

Sub. Code: CMT-2001

Core Sub. 1: Algebra- 2

Course Outcomes:

Upon completion of the course student will be able to

CO1. List and understand advance concepts of Algebra.

CO2. Identify, define and perform operations on modules.

CO3. Define and verify automorphisms and homomorphism of modules.

Unit 1

Division ring and Field, Extension fields, algebraic and transcendental extensions, Splitting fields, Normal extensions, Multiple roots, Finite fields, Separable extensions.

Unit 2

Automorphism fixed fields, Galois extension, Fundamental theorem of Galois Theory, Fundamental theorem of Algebra.

Unit 3

Modules (Definitions and examples), Submodules and Operation on modules

Unit 4

Homomorphisms of modules and quotient modules, completely reducible modules, finitely generated modules.

Reference Books:

- 1) P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Second Edition, Cambridge University Press, 1995.
- 2) M. Artin, Algebra, Prentice-Hall of India Private Ltd., New Delhi, 1994.
- 3) J. A. Gallian, Contemporary Abstract Algebra, Fourth Edition, Narosa Publishing House, New Delhi, 1999.
- 4) N. S. Gopalakrishnan, University Algebra, New Age International Private Ltd. Publishers, New Delhi, Sixth Reprint, 1998.
- 5) I. N. Herstein, Topics in Algebra, Second Edition, Wiley Pub. , New York, 1975.

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M.Sc. SEMESTER 2

Sub. Code: CMT-2002

Core Sub. 2: Complex Analysis

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Understand the concept of complex plane and generalize the concept of coordinate plane.
- CO2.** Determine continuity/differentiability/analyticity of a complex function and find the derivative of a function.
- CO3.** Evaluate a contour integral using parameterization, fundamental theorem of calculus and Cauchy's integral formula.
- CO4.** Compute the residue of a function and use the residue theory to evaluate a contour integral or an integral over the real line.
- CO5.** Analyze and classify the singularities of complex function in given region.

Unit 1

The extended complex plane and its spherical representation, analytic functions, bilinear transformations, their properties and classifications, Branches of many valued functions with special reference to $\arg z$, $\log z$ and z^a , elementary Riemann surfaces, definition and properties of conformal mapping.

Unit 2

Riemann – Steiltjes integral and its properties, line integral and its properties, fundamental theorem of calculus for line integral, Leibnitz rule, Taylor's theorem, Cauchy's integral formula and Cauchy's theorem for analytic functions on an open disc, winding number of a closed rectifiable curve with respect to a point outside the curve and its properties, Cauchy's integral formula first version and second version, Cauchy's theorem first version, second version, third version and forth version.

Unit 3

Cauchy – Goursat theorem, Moreras theorem, Cauchy's inequality, entire functions, Liouville's theorem, identity theorem, fundamental theorem of algebra, maximum modulus theorem and minimum modulus theorem.

Unit 4

Schwartz lemma, meromorphic functions, argument principle, Rouché's theorem, Open Mapping Theorem, Inverse function theorem.

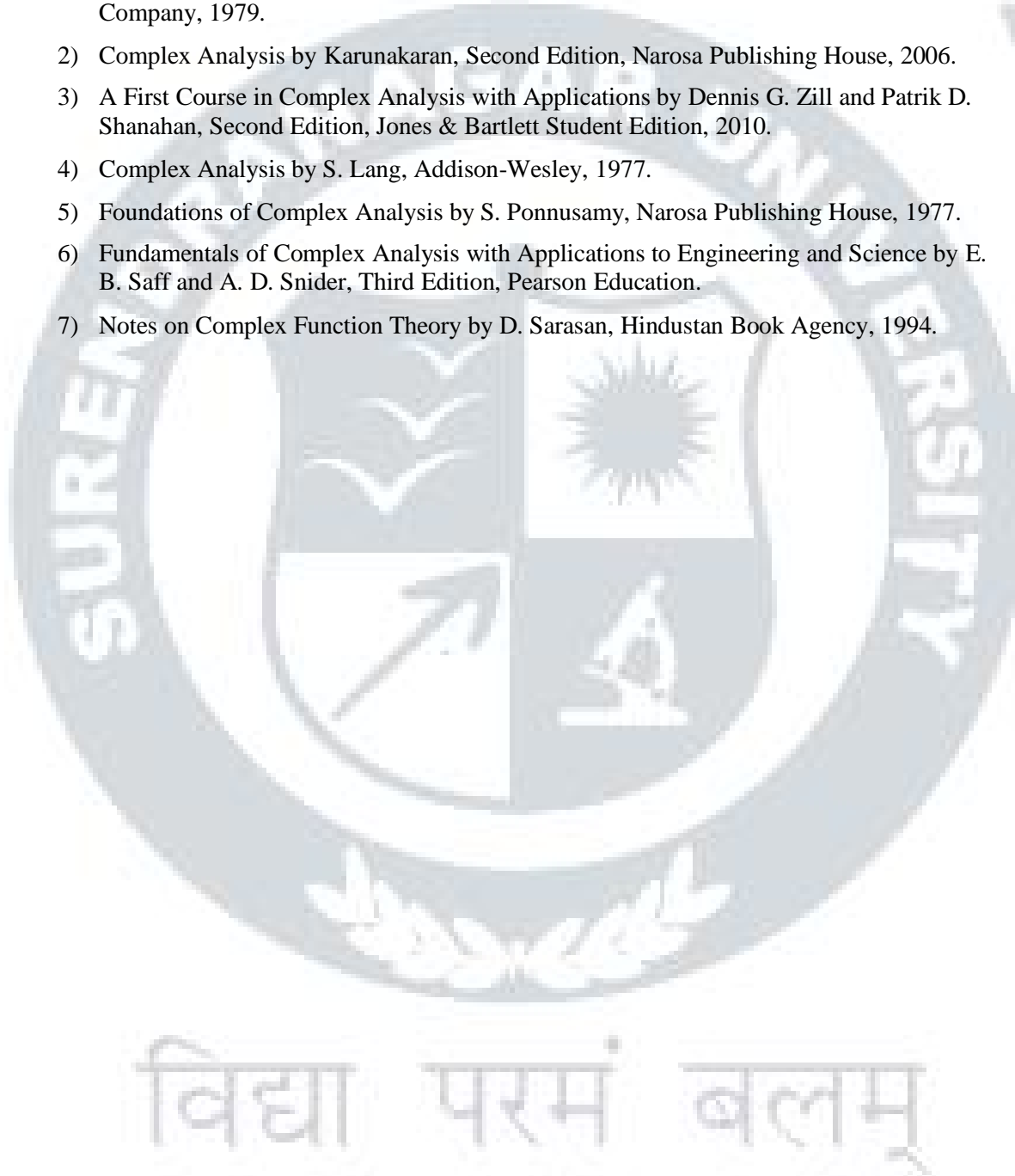
Unit 5

Isolated singularities, classifications of singularities, Laurent's series, residue theorem, evaluation of integrals.

This course is covered by relevant portions from the text “**Functions of One Complex Variable**” by John B. Conway, Third Edition, Springer International Student Edition, Narosa Publishing House.

Reference Books:-

- 1) Complex Analysis by L. V. Ahlfors, International Student Edition, Mc Graw – Hill Book Company, 1979.
- 2) Complex Analysis by Karunakaran, Second Edition, Narosa Publishing House, 2006.
- 3) A First Course in Complex Analysis with Applications by Dennis G. Zill and Patrik D. Shanahan, Second Edition, Jones & Bartlett Student Edition, 2010.
- 4) Complex Analysis by S. Lang, Addison-Wesley, 1977.
- 5) Foundations of Complex Analysis by S. Ponnusamy, Narosa Publishing House, 1977.
- 6) Fundamentals of Complex Analysis with Applications to Engineering and Science by E. B. Saff and A. D. Snider, Third Edition, Pearson Education.
- 7) Notes on Complex Function Theory by D. Sarasan, Hindustan Book Agency, 1994.



M.Sc. SEMESTER 2

Sub. Code: CMT-2003

Core Sub. 3: **Topology- 2**

Course Outcomes:

Upon completion of the course student will be able to

CO1. Understand, define and verify connectedness of topological spaces.

CO2. Understand, define and verify nets and filters.

CO3. State and prove the Tychonoff's theorem.

CO4. List, compare and classify the separation axioms of topological spaces.

CO5. Understand, define and verify concept of compact spaces.

Unit – 1:

Separation Axioms: T_1 – Spaces, T_2 – Spaces (Hausdorff Spaces).

Unit – 2:

Separation Axioms: Regular Spaces, Completely Regular Spaces, Normal Spaces.

Unit – 3:

Compact Spaces, Locally Compact Spaces, Limit Point Compact Spaces.

Unit – 4:

Sequentially Compact Spaces, Compact Metric Spaces.

Unit – 5:

Complete Metric Spaces.

Reference Books:-

- 1) Topology – A First Course, J.R.Munkres, Prentice Hall of India (2000). Chapter 3 (Article no. 26 to 29), Chapter 4 (Article no. 31,32,33 and 35) and Chapter 7 (Article no. 43)
- 2) General Topology by S.Willard, Addison – Wesley Publishing Company (1970)
- 3) Introduction to Topology & Modern Analysis, G.F.Simons, Tata Mcgraw Hill (2004)

M.Sc. SEMESTER 2

Sub. Code: CMT-2004

Core Sub. 4: Methods in Partial Differential Equations

Course Outcomes:

Upon completion of the course student will be able to

CO1. Identify and understand the higher order partial differential equations.

CO2. Understand and utilize the methods to solve the given partial differential equations

CO3. Understand and solve the given Boundary value problems and Equipotential surfaces.

Unit 1

Surfaces and Curves in three dimensions, Simultaneous differential equations of the first order and the first degree in three variables, Methods of solutions of $dx/P = dy/Q = dz/R$, Orthogonal trajectories of a system of curves on a surface. Pfaffian Differential forms and equations, Solution of Pfaffian differential equations in three variables, and Miscellaneous problems.

Unit 2

Partial differential equations, Origins of First-order partial differential equations, Linear equations of the first order, Integral Surfaces passing through a given curve, Surfaces orthogonal to a given system of surfaces.

Unit 3

Non-linear partial differential equations of the first order, Charpit's method, Special types of first order equations, Solutions satisfying the given conditions, Jacobi's method, and Miscellaneous problems.

Unit 4

The origin of second order equations, Linear partial differential equations with constant coefficients, and Equations with variable coefficients.

This course is covered by the relevant portions from the book 'Elements of Partial Differential Equations' by Ian Sneddon, McGraw-Hill Book Company.

Reference Books:-

1. Partial Differential Equations by F. John, Narosa Publishing Company, New Delhi, 1979.
2. Elementary Course in Partial Differential Equations by Amarnath, Narosa Publishing House, New Delhi, 1997.

M.Sc. SEMESTER 2

Sub. Code: EMT-2001

Elective Sub. 1: **Classical Mechanics -2**

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Understand, define and verify Rigid Body Equations of Motion.
- CO2.** Understand and compare theory of relativity in classical mechanics.
- CO3.** Derive the Hamilton's equation of motion.
- CO4.** Understand and utilize the Canonical transformations and Generating functions.

Unit 1: The Rigid Body Equations of Motion

Angular momentum and kinetic energy of motion about a point, the inertia tensor and moment of inertia, the heavy symmetrical top with one point fixed.

Unit 2: Special Relativity in Classical Mechanics

The basic program of special relativity, The Lorentz transformation, Lorentz transformations in real four dimensional spaces, Further descriptions of the Lorentz transformation, Covariant four – dimensional formulations, The force and energy equations in relativistic mechanics.

Unit 3: Hamilton's equation of Motion

Derivation of Hamilton's equation of motion, Routh's procedure, derivation of Hamilton's equation from Hamilton's Principle, principle of least action, problem related to above topics.

Unit 4: Canonical transformations and Generating functions

Poisson's brackets and their properties, Hamilton-Jacobi theory, problem related to above topics.

The course is covered by the above topics from the book:

1. **Classical Mechanics** by H. Goldstein, 2nd Edition, Narosa Publishing House
2. **Classical Mechnaics** by C. R. Mondal, Prentice Hall of India Pvt. Ltd.

M.Sc. SEMESTER 3

Sub. Code: **CMT-3001**

Core Sub. 1: **Prog. In C & Numerical Methods**

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Apply suitable and effective methods called Numerical Methods, for obtaining approximate representative numerical results of the problems.
- CO2.** Solve problems in the field of Applied Mathematics, Theoretical Physics and Engineering which requires computing of numerical results using certain raw data.
- CO3.** Develop logics which will help them to create programs, applications in C.
- CO4.** By learning the basic programming construction, they can easily switch over to any other language in future.

Unit 1

Constants, variables, C tokens, keywords, identifiers, declaration of variables, operations and expressions, managing input and output operations and formatted output.

Unit 2

Decision making and branching statements like – if then else, if then switch, go to and loops, jump in loops

Unit 3

One or two dimensional array and their initialization, handling of character strings, User defined functions, structure, unions, pointers and file management in C.

Unit 4

Iterative methods introduction, beginning an iterative method, method of successive bisection, method of false position, Newton-Raphson iterative method, secant method, method of successive approximation, comparison of iterative methods, solution of polynomial equation.

Unit 5

Solution of simultaneous algebraic equations introduction, Gauss elimination method, ill conditioned equations, refinement of the solution obtained by Gaussian elimination, Gauss-Seidel iterative method, comparison of direct and iterative methods. Interpolation introduction, Lagrange interpolation, difference tables.

Reference Books:-

1. Introductory methods of Numerical analysis by S S Sastry, Prentice Hall of India, 1998.
2. Computer Oriented Numerical Methods by V. Rajaraman, Prentice Hall of India, 1994.
3. Programming in C, by E. Balagurusami units 2 to 12.

M.Sc. SEMESTER 3

Sub. Code: CMT-3002

Core Sub. 2: **Functional Analysis**

Course Outcomes:

Upon completion of the course student will be able to

CO1. Understand the concept of Normed Linear Spaces and Banach Spaces.

CO2. Classify the weak and strong convergence of sequences.

CO3. State and prove uniform boundedness theorem.

CO4. Understand the structures of Inner Product Spaces and Hilbert Spaces.

CO5. State and Prove the Hahn-Banach Theorem.

Unit 1

Normed linear spaces, Banach spaces, Quotient space of a normed linear spaces and its completeness, bounded linear transformations, normed linear spaces of bounded linear transformations, dual spaces with examples.

Unit 2

Weak convergence in normed linear spaces, equivalent norms, Riesz lemma, Basic properties of finite dimensional normed linear spaces and compactness, weak convergence in normed linear spaces, reflexive spaces.

Unit 3

Uniform Boundedness theorem and its consequences, open mapping theorem, closed graph theorem, Hahn-Banach theorem for normed linear spaces, compact operations, solvability of linear equations in Banach spaces, the closed range theorem.

Unit 4

Inner product space, Hilbert space, orthonormal sets, Bessel's inequality, complete orthonormal sets, Parseval's identity, structure of Hilbert spaces, projection theorem, Riesz representation theorem for bounded linear functional on Hilbert spaces, reflexivity of Hilbert spaces.

Unit 5

Adjoint of an operator on a Hilbert space, self – adjoint, Normal, Unitary, Positive and Projection operators on Hilbert spaces, abstract variation boundary – value problem, the generalized Lax-Milgrem theorem.

This course is covered by relevant portions from the text “**Introductory Functional Analysis with Applications**”, John Wiley and Sons, Newyork, 1978.

Reference Books:-

1. Bachman G. and Warici L, Functional Analysis, Academic Press, 1966.
2. Conway J. B., A Course in Functional Analysis, Springer-verlag, Newyork, 1990.
3. Krishnan V. K. , Text Book of Functional Analysis; A Problem oriented approach, Printice Hall of India, 2001.

4. Limaye B. V., Functional Analysis, New Age International Pvt. Ltd., 2001.
5. Simmons G. F., Introduction to Topology and Modern Analysis, McGraw – Hill book company, Newyork, 1963.
6. Tayor A. E., Introduction to Functional analysis, John Wiley and Sons, Newyork, 1958.



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M.Sc. SEMESTER 3
Sub. Code: CMT-3003
Core Sub. 3: Number Theory – 1

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Understand the basic concepts of number theory.
- CO2.** Recognize and identify the properties of prime numbers.
- CO3.** Understand the concepts of congruences.
- CO4.** Utilize the concept of combinatorial number theory.
- CO5.** Construct mathematical proofs of statements and find counterexamples to false statements in number theory.

Unit 1

Divisibility, Prime Numbers.

Unit 2

Congruences, Linear Congruences and their solutions, Chinese Remainder Theorem, Degree of a Congruence relation and related theorems.

Unit 3

Primitive rules and related Theorems and Examples, Related Congruences and their solutions.

Unit 4

Largest Integer functions and related results, Arithmetic Functions.

Reference Books:-

1. THE THEORY OF NUMBERS (Authors: Ivan Niven ,Herbert S. Zuckerman, Hugh L. Montgomery)
2. NUMBER THEORY (Authors: Z. I. Borevich and I. R. Shafarevich)
3. AN INTRODUCTION TO THE GEOMETRY OF NUMBERS (Authors: J. W. S. Cassels)
4. HISTORY OF THE THEORY OF NUMBERS (Authors: L. E. Dickson)

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M.Sc. SEMESTER 3

Sub. Code: **CMT-3004**

Core Sub. 3: **Discrete Mathematics**

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Understand the algebraic structures including semigroups and monoids.
- CO2.** State and prove basic results of homomorphism between semigroups.
- CO3.** Understand the concept of Boolean algebra and derive related results.
- CO4.** Understand and apply the finite state machine and coding theory.

Unit 1

Semigroups and Monoids, Homomorphism of Semigroups and Monoids, Products and Quotients of semigroups, Fundamental theorem of Homomorphism of Semigroups, Subsemigroups and submonoids. Relations, Transitive Closure and Warshall's Algorithm

Unit 2

Lattices as partially ordered sets, Properties of Lattices, Lattices as algebraic systems, Sublattices, Direct product and Homomorphisms of Lattices, Some Special Lattices, Finite Boolean Algebras, Functions on Boolean Algebras, Karnaugh Map Method.

Unit 3

Languages and Grammars, Finite State Machines, Semigroups, Machines and Languages, Moore Machines, Simplification of Machines, Moore Machines and Regular Languages, Kleene's Theorem, Pumping Lemma, Nondeterministic Finite State Automata.

Unit 4

Propositions and Logical operations, Truth tables, Conditional statements and Logical Equivalence, Quantifiers, Rules of Inference.

Unit 5

Elements of Coding Theory, The Hamming Metric, The Parity-Check and Generator Matrices, Group Codes: Decoding with Coset Leaders, Hamming Matrices.

Reference Books:-

1. Grimaldi, R.P, Discrete and Combinatorial Mathematics, 3rd Edition, Addison-Wesley Publishing Company, 1994.
2. Johnsonbaugh, R., Discrete Mathematics, Pearson Education, First Indian Reprint, 2001.
3. Kolman, B, Busby, R.C., Ross, S.C., Discrete Mathematical Structures, 5th Edition, Pearson Education, 2006.
4. Lawson, M.V., Finite Automata, Chapman and Hall/CRC Press, 2004.
5. Tremblay, J.P., Manohar, R., Discrete Mathematical Structures with Applications to Computer Science, Tata-McGraw Hill Publishing Company Limited, New Delhi, 21st Reprint, 2004.

M.Sc. SEMESTER 3

Sub. Code: EMT-3011

Elective Sub. 1: Differential Geometry

Course Outcomes:

Upon completion of the course student will be able to

CO1. Understand and define the curves and surfaces.

CO2. Understand the concepts of curvature, torsion, tangent, normal and binormal

CO3. Prove Frenet – Serret theorem.

CO4. Derive the formulae for first and second fundamental forms.

Unit 1

Local theory of curves, space curves, examples. Planar curves, Helices, Frenet – Serret apparatus. Existence of space curves, involutes and evolutes of curves.

Unit 2

Local theory of surfaces – parametric patches on surface. First Fundamental form and arc length.

Unit 3

Normal curvature, Geodesic curvature and Gauss formulae, Shape operator L^p of a surface at a point, vector field a curve.

Unit 4

Second and third fundamental forms of a surface, Weingarten map, principal curvatures, Gaussian curvature, mean and normal curvatures.

Unit 5

Riemannian curvatures, Gauss theorem of Egregium, isometry groups and fundamental existence theorem for surfaces.

Reference Books:-

1. R. S. Milman and G. D. Parker, Elements of Differential Geometry, Prentice – Hall, 1977.
2. B. O' Neil, Elements of Differential Geometry, Academic Press, 1966.
3. M. Docarmo, Differential Geometry of curves and surfaces, Prentice – Hall, 1976.
4. J. A. Thorpe, Introduction to Differential Geometry, Springer – Verlag.
5. S. Sternberg, Lecture notes on Differential Geometry, Prentice – Hall, 1964.

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M.Sc. SEMESTER 3

Sub. Code: **EMT-3021**

Elective Sub. 2: **Special Theory of Relativity and Tensor Analysis**

Course Outcomes:

Upon completion of the course student will be able to

CO1. Use tensor notation in relativity theory.

CO2. Apply the concepts of length contraction and time dilation as well as use Lorentz transformations.

CO3. Analyze Einstein's field equations as well as know and use some important solutions to these.

CO4. Report some experimental tests of general relativity.

CO5. Have knowledge about cosmological models.

Unit 1

- ☐ Newtonian Relativity (Galilean Transformation)
- ☐ Lorentz transformation
- ☐ Michelson – Morley experiment

Unit 2

- ☐ Length Contraction
- ☐ Time dilation
- ☐ Relativistic law of addition of velocities
- ☐ Equivalence of mass and energy
- ☐ Problems related to above topic

Unit 3

- ☐ Tensor Algebra
- ☐ Vector field in affine and Riemann space

Unit 4

- ☐ Christoffel Symbols
- ☐ Tensor Analysis

Books:-

1. Related topics of Unit 1 and Unit 2 will be covered from the book “**Special Relativity**” by W. Rindler. Pub.: Oliver and Bosed.
2. Related topics of Unit 3 and Unit 4 will be covered from the book “**Introduction to General Relativity**” by R. Adler, M. Basin, M. Schiffer. Pub.: Mc.Graw Hill Kogakusha Ltd.

Reference Books:-

1. The Special theory of Relativity – Benerji and Benarjee. Pub.: Prentice Hall India Ltd.
2. Essential Relativity – W. Rindler. Pub. Springer Verlag.

M.Sc. SEMESTER 4

Sub. Code: **CMT – 4001**
Core Sub. 1: **Linear Algebra**

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Understand the concepts of linear algebra including transformations and canonical transformations.
- CO2.** State, prove and apply the Cayley-Hamilton theorem
- CO3.** Analyze and select proper methods to solve a given system of linear equations
- CO4.** Understand and utilize the Sylvester's law of inertia.
- CO5.** Understand the concept of bilinear and quadratic forms.

Unit 1

The Algebra of linear transformations, Characteristic roots, Matrices.

Unit 2

Canonical Forms: Triangular Form, Nilpotent linear transformations, Invariants of a nilpotent linear transformation.

Unit 3

Canonical Forms: The primary decomposition theorem, Jordan Form, Rational canonical Form.

Unit 4

Trace and Transpose, Determinants, Cramer's rule, Cayley-Hamilton theorem, a quick review of inner product spaces, Hermitian, Unitary and Normal transformations.

Unit 5

Real Quadratic Forms, Sylvester's law of inertia, Bilinear Forms, Symmetric Bilinear Forms, Skew-Symmetric Bilinear Forms, Groups preserving Bilinear Forms.

Reference Books:-

1. N. Herstein, **Topics in Algebra**, 2/e, Wiley Publication, 1975. (For Unit 1 to Unit 4)
2. K. Hoffman & R. Kunze, **Linear Algebra**, 2/e, Prentice Hall of India, New Delhi, 1992. (For Unit 5)

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M.Sc. SEMESTER 4

Sub. Code: **CMT-4002**

Core Sub. 2: **Integration Theory**

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Define and understand basic notions in abstract integration theory, integration theory on topological spaces and the n-dimensional space
- CO2.** Describe and apply the notion of measurable functions and sets and use Lebesgue monotone and dominated convergence.
- CO3.** Develop an appreciation of the basic concepts of measure theory.
- CO4.** These methods will be useful for further study in a range of other fields, e.g. Stochastic calculus, Quantum Theory and Harmonic analysis.

Unit 1

Measures spaces, Measurable functions, integration, general convergence theorems.

Unit 2

Signed measures, Positive sets, negative sets, null sets and their properties, Hahn-Decomposition Theorem, mutually singular measures, Jordan-Decomposition for a signed measure.

Unit 3

Measure absolutely continuous with respect to another measure, Radon-Nikodym theorem for measure and for signed measure, Lebesgue decomposition theorem, outer measure on a set, algebra of sets, Caratheodary extension theorem.

Unit 4

Product measure, structure of measurable sets in the product measure space, Fubini's theorem, Fonelli's theorem, and Riesz Representation theorem for bounded linear functional on , Baire measure on the real line, Lebesgue Stieltjes integral of Borel measurable function with respect to monotonically increasing function.

Unit 5

Locally compact Hausdorff spaces, Baire and Boral measures, continuous functions with compact support, regularity of measures on locally compact Hausdorff spaces, integration of continuous functions with compact support, Riesz Markov-theorem.

Reference Books:-

1. H. L. Royden, Printice Hall of India, Third edition, 1987.
2. G. de Barre, Measure Theory and Integration, Wiley Eastern Limited, 1981.
3. P. R. Halmos, Measure Theory, Van Nostrand, Princeton, 1950.
4. W. Rudin, Real and complex analysis, Tata McGraw Hill Publishing Company, Second Edition, 1974.
5. S. K. Berberian, Measure and Integration, Chelsa Publishing Company, Newyork, 1965.
6. K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, 1997.

M.Sc. SEMESTER 4

Sub. Code: CMT-4003

Core Sub. 3: Number Theory – 2

Course Outcomes:

Upon completion of the course student will be able to

CO1. Understand, analyses and solve the Diophantine Equations.

CO2. Approximate Irrationals by Rationales.

CO3. State and prove the Hurwitz's Theorem.

CO4. Understand the concepts of partition function and Ferrers graphs.

Unit 1

Farey Fractions, Irrational numbers, Farey Fractions and Approximation of Irrationals by Rationals.

Unit 2

Continued Fractions(Finite and Infinite), Approximations of Irrationals by Rationals, Hurwitz's Theorem.

Unit 3

Periodic Continued Fractions, Pell's Equations.

Unit 4

Diophantine Equations, Pythagorean Triplets, Some other Examples.

Reference Books:-

1. THE THEORY OF NUMBERS (Authors: Ivan Niven ,Herbert S. Zuckerman, Hugh L. Montgomery)
2. NUMBER THEORY (Authors: Z. I. Borevich and I. R. Shafarevich)
3. AN INTRODUCTION TO THE GEOMETRY OF NUMBERS (Authors: J. W. S. Cassels)
4. HISTORY OF THE THEORY OF NUMBERS(Authors: L. E. Dickson)

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M.Sc. SEMESTER 4

Sub. Code: **CMT-4004**
Core Sub. 4: **Graph Theory**

Course Outcomes:

Upon completion of the course students will be able to

- CO1.** Understand the fundamental concepts of graphs.
- CO2.** Characterize the Euler and Hamiltonian Graphs.
- CO3.** Understand and apply the Kruskal's and Prim's algorithm.
- CO4.** Determine the planarity of the given graph.
- CO5.** Understand the concept of graph coloring.

Unit 1

Graph, degree of a vertex, path, circuit, connected and disconnected graphs, components, adjacency and incidence matrix.

Unit 2

Euler circuits, Euler graph, Hamiltonian Paths and circuits.

Unit 3

Trees and their characterizations, Cut-Sets and Cut-Vertices

Unit 4

Planar Graphs, Kuratowski's two graphs, Different representation of planarity, Detection of Planarity.

Unit 5

Coloring of graphs, chromatic number, chromatic polynomial, the four color problem matching

Unit 6

Graph theory in Operation Research: transport networks, extension of Max-Flow, Min-Cut theorem, minimal cost flows.

The syllabus is covered from chapters 1 & 2 (for quick review), Chapter 3 (3.1 to 3.6), 4 (4.1 to 4.6), 5 (5.1 to 5.5), 8 (8.1 to 8.4) and 14(14.1 to 14.3) from **“Graph theory with application to Engineering & computer science”** by Narsingh Deo, Prentice – Hall of India New Delhi.

Reference Books:-

1. Graph theory by F. Harary, Addison – Wesley, 1969.
2. Graph Theory and Its application by J. Gross and J. Yellen, CRC Press, 2000.
3. Introduction to Graph Theory by D. B. West, 2/e, Prentices – Hall of India, 2001.
4. A textbook of Graph Theory by R. Balakrishnan and K. Ranganathan, Springer, 2012.

M.Sc. SEMESTER 4

Sub. Code: **EMT-4011**

Elective Sub. 1: **Financial Mathematics**

Course Outcomes:

Upon completion of the course studentt will be able to

CO1. Categorize the various financial markets including stock markets, currency market and bond markets.

CO2. Differentiate between options and contracts.

CO3. State and prove Ito's lemma.

CO4. State and prove Black – Sholes theorem.

Unit 1

Basic option theory, Types of options, interest rates and present value, Asset price

Unit 2

Random walk, Ito's lemma, Black-Sholes model, arbitrage theorem, option values

Unit 3

The Black – Sholes formulae, hedging the practice, partial differential equations and Black – Sholes formulae.

Unit 4

Variations in Black – Sholes model to include dividends as well as forward and future contracts, American Options.

Reference Books:-

1. P. Willmontt, S. Howison and J. Dewynne, the Mathematics of Financial Derivatives, Cambridge Univ. Press, 1995.
2. Sheldon M. Ross, An elementary introduction to Mathematical Finance, Cambridge Univ. Press, 2003.

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M.Sc. SEMESTER 4

Sub. Code: EMT-4021

Elective Sub. 2: General Theory of Relativity & Cosmology

Course Outcomes:

Upon completion of the course student will be able to

CO1. Know the fundamental principles of the general theory of relativity.

CO2. Know the meaning of basic concepts like the equivalence principles, inertial frames and how gravity is understood as a manifestation of a curved space-time.

CO3. Familiar with some of the main contents of the theory: motion in the gravitational field, time dilation and frequency shifts, bending of light, gravitational waves and cosmological models with expanding space.

Unit 1: The Gravitational Field Equation in Empty Space

- ☐ Criteria for the field equations.
- ☐ The Riemann curvature tensor and its properties.
- ☐ The Bianchi identities.

Unit 2: The Schwarzschild solution and its consequences, experimental tests of General Relativity

- ☐ The Schwarzschild solution
- ☐ The Schwarzschild solution in isotropic co-ordinates
- ☐ The General Relativistic Kepler problem and the perihelion shift of Mercury.
- ☐ The trajectory of light ray in Schwarzschild field.
- ☐ The Schwarzschild radius, Kruskal co-ordinates and the Black hole.

Unit 3: The Kerr Solution

- ☐ The Schwarzschild and Kerr solution
- ☐ The Kerr solution and Rotation.

Relevant topics will be covered from “**Introduction to General Relativity**”. – By R. Adees, M. Bazin & M. Schiffer.

Reference Books:-

1. Essential Relativity – W. Rindler. Pub.: Springer Verlag
2. General Relativity and Cosmology – J. V. Narlikar, Mc-Millan India Ltd.
3. An Introduction to Cosmology – J. V. Narlikar, 3rd edition, Cambridge University Press.

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M.Sc. SEMESTER 4

Sub. Code: EMT – 4031

Elective Sub. 3: **Commutative Ring Theory**

Course Outcomes:

Upon completion of the course students will be able to

- CO1.** Know basic definitions concerning elements in rings, classes of rings, and ideals in commutative rings.
- CO2.** Know constructions like tensor product and localization, and the basic theory for this.
- CO3.** Know basic theory for noetherian rings and Hilbert basis theorem.
- CO4.** Know basic theory for integral dependence, and the Noether normalization lemma.
- CO5.** Know the interplay between ideals in polynomial rings, and the corresponding geometric objects: affine varieties.

Unit 1

Rings and ring homomorphisms, Ideals, Quotient rings, Zero-divisors, Nilpotent elements, Units, Prime ideals and Maximal ideals, Nilradical and Jacobson radical, Operations on ideals, Extension and contraction.

Unit 2

Modules and module homomorphisms, Submodules and quotient modules, Operation on Submodules, Direct sum and product, Finitely generated modules, Exact sequences, Rings and modules of fractions, Local properties, Extended and contracted ideals in rings of fractions.

Unit 3

Primary ideals, Primary decomposition, First uniqueness theorem, Second uniqueness theorem, Integral dependence, The Going-Up theorem, Integrally closed integral domains, The Going-Down theorem, Valuation rings.

Unit 4

Noetherian modules, Artinian modules, Composition series of a module, Noetherian rings, Hilbert's basis theorem, Primary decomposition in Noetherian rings.

Unit 5

Artin rings, Structure theorem for Artin rings, Discrete Valuation rings, Dedekind domains, fractional ideals.

Text Book:-

Introduction to Commutative Algebra by M. F. Atiyah and I. G. Macdonald, Addison-Wesley Publishing Company, 1969. (Chapter 1 to Chapter 9)

Reference Books:-

1. N. Bourbaki, **Commutative Algebra**, Springer – Verlag, New York, 1985.
2. O. Zariski & P. Samuel, **Commutative Algebra** Volume I, Van Nostrand, Princeton, 1958.

3. D. G. Northcott, **Lessons on rings, modules and multiplicities**, Cambridge University Press, 1968.
4. D. Eisenbud, **Commutative Algebra with a view toward Algebraic Geometry**, Graduate Texts in Mathematics 150, Springer – Verlag, New York, 1995.



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M.Sc. SEMESTER 4

Sub. Code: EMT – 4041

Elective Sub. 4: Introduction to Mathematical Cryptography

Course Outcomes:

Upon completion of the course student will be able to

- CO1.** Explain the idea of public-key cryptography and the common algorithms used.
- CO2.** Describe the basic issues around finding large prime numbers and factoring large composite numbers, including various techniques for both.
- CO3.** Explain the significance of these problems to public-key cryptography.
- CO4.** Define elliptic curves and explain the group law on these curves, both geometrically and formulaically.
- CO5.** Explain how elliptic curves are used in certain cryptographic algorithms.
- CO6.** Explain and use basic theorems about arithmetic in the ring's \mathbb{Z}/n , the theory of finite abelian group, and elliptic curves.

Unit 1

Modular arithmetic, the language of rings and fields, finding multiplicative inverses in \mathbb{Z}/n , Fermat's little theorem, the primitive root theorem for \mathbb{Z}/p .

Unit 2

The basic idea of public key cryptography, Diffie – Hellman key exchange and the ElGamal cryptosystem. Language for measuring the complexity of algorithms, and lengths of running times. Attempts to break codes by solving the Discrete Logarithm Problem: brute force attacks, the collision method, and the Pohlig - Hellman algorithm. The Chinese Remainder Theorem.

Unit 3

Euler's formula for powers in \mathbb{Z}/n , and the RSA cryptosystem. How to find large primes: the Prime Number Theorem and some Monte Carlo Methods (e.g. the Miller-Rabin test). Algorithms for factoring large integers: Pollards ρ algorithm.

Unit 4

Elliptic curves. Smoothness, the point at infinity, the group law. Using elliptic curves for cryptography. Classification of finite abelian groups. Integer factorization using elliptic curves (Lenstra's method).

Text Book:-

An Introduction to Mathematical Cryptography by Jeffrey Hoffstein, Jill Pipher & Joseph H. Silverman, Springer – Verlag, 2008. (Chapters 1, 2, 3 & 5)

Reference Books:-

1. Paul Garrett, **Making, Breaking Codes: Introduction to Cryptology**, 1/e, Prentice Hall, (2000).
2. Douglas Stinson, **Cryptography: Theory and Practice**, 2/e, Chapman & Hall/CRC, (2002).
3. J. H. Silverman, **A friendly introduction to number theory**, Prentice Hall, (2001).
4. J. Menezes, P. C. Van Oorschot & S. A. Vanstone, **The handbook of Applied Cryptography**, CRC Press, (1996).
5. Neal Noblitz, **Algebraic Aspects of Cryptography**, Springer, (1998).
6. J. A. Buchmann, **Introduction to Cryptography**, Springer – Verlag, (2000).